Generating Readable Diagrammatic Proofs

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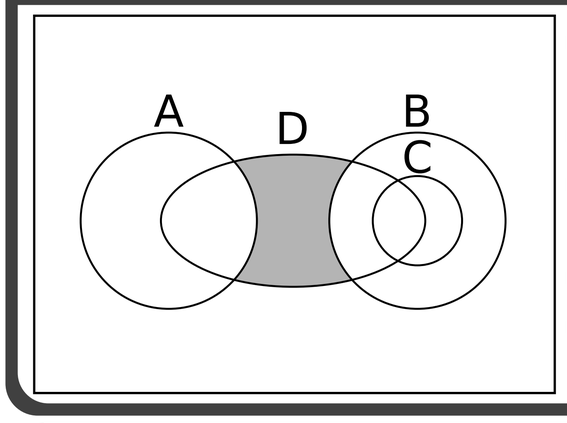
Formal Euler Diagrams



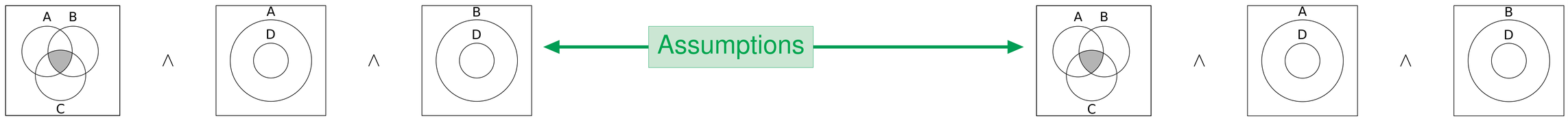
- Define set-theoretic properties visually
- Curves: sets
- Topology: relations between sets
- Shading: emptiness
- May be composed with Boolean operators

Proof Rules

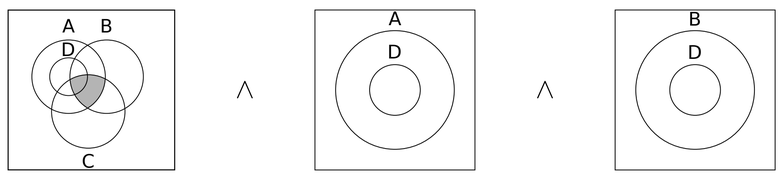
- Transfer information between diagrams
- Change topology within a diagram
- Proven to be correct
- Premises imply antecedent
- Can be applied to elements within compound diagram

$$\frac{\text{Premiss} \quad \wedge \quad \text{Premiss}}{\text{Antecedent}} \qquad \frac{\text{Premiss}}{\text{Antecedent}}$$

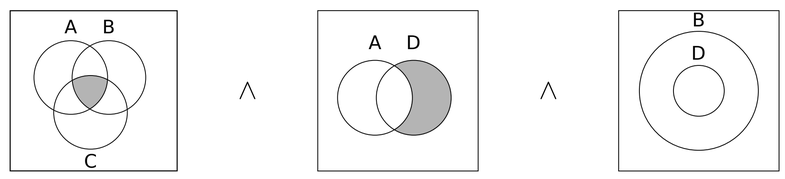
Proving Theorems with Euler Diagrams



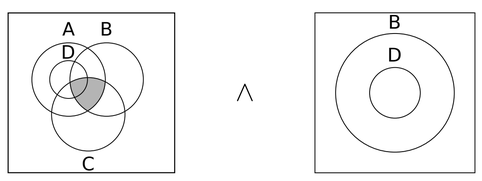
Copy Contour ⊢



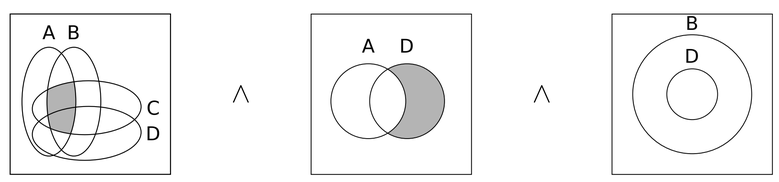
Introduce Shaded Zone ⊢



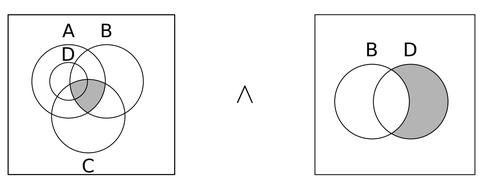
Remove Conjunct ⊢



Introduce Contour ⊢

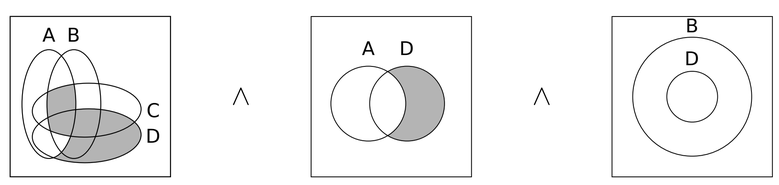


Introduce Shaded Zone ⊢



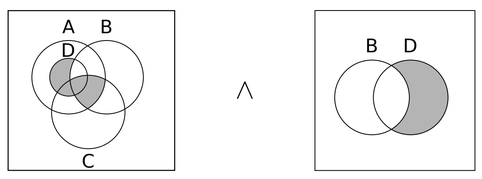
Type of Rules

Copy Shading ⊢



Application of proof rules

Copy Shading ⊢



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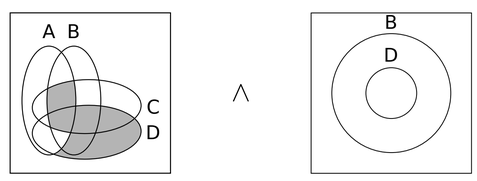

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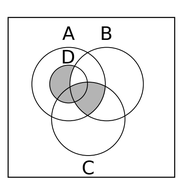
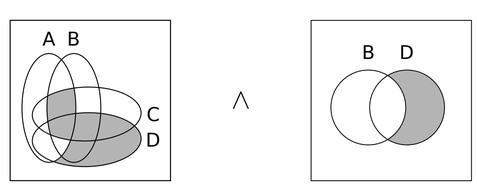
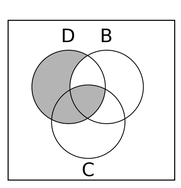


Introduce Shaded Zone ⊢

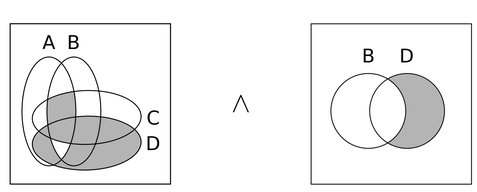


Amount of Clutter

Erase Contour ⊢

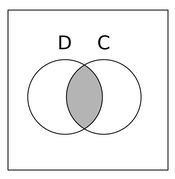


Copy Shading ⊢

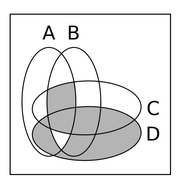


Which of these proofs is more readable?

Erase Contour ⊢

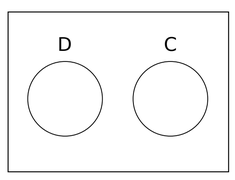


Remove Conjunct ⊢



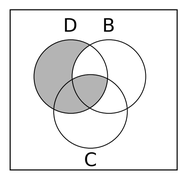
Conclusion

Remove Shaded Zone ⊢

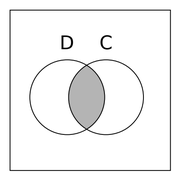


Length

Erase Contour ⊢

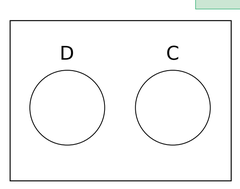


Erase Contour ⊢



Conclusion

Remove Shaded Zone ⊢



Strategies

- Identify conditions for easier understanding of rule applications
- Empirical study examining user errors while identifying rules
- Derive different strategies from study
  - Equalise sets of zones?
  - Minimise length?
- Heuristics for different user preferences
- Implementation within Speedith [1]
- User guidance for readable proofs
- Automated heuristical proof search

[1] Matej Urbas, Mateja Jamnik, Gem Stapleton, and Jean Flower. Speedith: A diagrammatic reasoner for spider diagrams. In *Diagrammatic Representation and Inference*, pages 163–177. Springer, 2012.

This work is funded by EPSRC grant EP/M011763/1 *The readability of proofs in diagrammatic logic*